

# Inflationary Constraints on Late Time Modulus Dominated Cosmology

KOUSHIK DUTTA\* and ANSHUMAN MAHARANA†

*\*Theory Division  
Saha Institute of Nuclear Physics  
1/AF Salt Lake  
Kolkata, 700064, India*

*†Harish Chandra Research Institute  
Chattnag Road, Jhansi  
Allahabad, 211019, India*

## Abstract

We consider cosmological scenarios in which density perturbations are generated by the quantum fluctuations of the inflaton field at early times; the late time dynamics involves a modulus which first dominates the energy density of the universe and then decays to reheat the visible sector. By examining the evolution of energy density of the universe from the time of horizon exit of a pivot mode to the present day, and the fact that a modulus field decays via Planck suppressed interactions, we arrive at a relation which relates the mass of the modulus, inflationary observables/parameters and broad characteristics of the post inflationary reheating phase. When viewed together with generic expectations regarding reheating and the initial field displacement of the modulus after inflation, the relation gives a bound on the minimum mass of the modulus. For a large class of models, the bounds obtained (for the central values of Planck data) can be much stronger than the “cosmological moduli problem” bound.

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Email: \*koushik.dutta@saha.ac.in, †anshumanmaharana@hri.res.in

# 1 Introduction

The hot big bang model together with the inflationary paradigm provides a highly attractive framework for cosmology. Typically, it is assumed that after inflation the visible sector degrees of freedom reheat and have evolved adiabatically since then. Baryon asymmetry is tied to high scale physics via mechanisms such as leptogenesis and dark matter is a thermal relic. In spite of the impressive successes of this scenario it is important to keep in mind that the big bang model is consistent with entropy production as long as this took place before nucleosynthesis.

In fact, the cosmology in many beyond the standard model scenarios can potentially involve late time entropy production as a result of decay of long lived light scalar fields (light moduli in the context of supergravity/string models). Light moduli are typically displaced from their minimum during inflation. At the end of inflation the universe reheats, energy associated with the inflaton gets converted to radiation. As the universe expands the Hubble constant decreases, when it becomes of the order of the mass of a modulus the modulus begins to oscillate about the minimum of its potential. Subsequently, the energy density associated with the field begins to redshift like matter, at a rate significantly slower than the radiation – the energy associated with the modulus can quickly dominate the energy density of the universe. Eventually the modulus decays reheating the universe.

The last modulus to decay<sup>1</sup> essentially provides the “initial conditions” for cosmological evolution. The reheat temperature is given by the decay width of the scalar field. The properties such as dark matter density, baryon asymmetry are determined by the branching ratio of the various decay products. This makes the scenario highly predictive for models where it is possible to compute the couplings of the scalar field to the standard model and other light degrees of freedom. This predictivity comes at a cost; the late time reheating typically washes out a lot of information making the connection to early universe physics very challenging.

In this paper we will obtain a relationship between the mass of the modulus, inflationary observables and broad characteristics of post inflationary reheating (number of e-foldings and the effective equation state). When viewed along with the usual expectations regarding the equation of state during reheating and the initial field displacement of the modulus, this immediately leads us to a lower bound on the mass of the modulus; cf. Eq. (3.15). For a large class of inflationary potentials, the bounds can be stronger

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<sup>1</sup> The non thermal matter dominated universe must end prior to big bang nucleosynthesis. We know with great confidence that at the time of primordial nucleosynthesis the universe was radiation dominated.

than those provided by the “cosmological moduli problem”. The constraint depends exponentially on the number of e-foldings between horizon exit and the end of inflation, future observations are going to be crucial in sharpening the limits.

This paper is organised as follows. In section 2 we discuss some basic aspects of the dynamics of cosmologically relevant scalars. We also briefly discuss the motivations for their appearance in string/supergravity models and some related phenomenological scenarios. In section 3, we present our analysis, obtain the bound and discuss implications. We conclude in section 4.

## 2 Cosmologically Relevant Scalars

As discussed in the introduction, long lived scalar fields with masses below the Hubble scale during inflation are expected to be cosmologically relevant. Here we give a brief review of some aspects that will be relevant for us and refer the reader to the seminal papers [1–3] for a complete discussion.

The cosmological evolution of a scalar field is given by

$$\ddot{\varphi} + (3H + \Gamma_{\varphi})\dot{\varphi} + \frac{\partial V(\varphi)}{\partial \varphi} = 0, \quad (2.1)$$

where  $H$  is the Hubble constant and  $\Gamma_{\varphi}$  the width of the scalar. For long lived scalars, this implies that the field is frozen at its initial displacement  $\varphi_{\text{in}}$  if  $H > m_{\varphi}$ . The initial displacement can be due to thermal/quantum fluctuations of the field during inflation [5] or explicit dependence of the potential for the modulus on the inflaton vev [6–9] and is expected to be of the order of  $M_{\text{pl}}$ . After the modulus begins to oscillate about its minimum; matter radiation equality takes place at total energy density

$$\rho_{\text{eq}} = m_{\varphi}^2 \varphi_{\text{in}}^2 \left( \frac{\varphi_{\text{in}}^2}{6M_{\text{pl}}^2} \right)^3. \quad (2.2)$$

The modulus then dominates the energy density of the universe. It decays at time  $t = \tau_{\text{mod}}$ , the corresponding energy density is

$$\rho_{\text{decay}} \sim M_{\text{pl}}^2 \Gamma_{\varphi}^2, \quad (2.3)$$

where  $\Gamma_{\varphi}$  is the total width of the field. Moduli fields interact only via Planck suppressed interactions, their lifetime

$$\tau_{\text{mod}} \approx \frac{1}{\Gamma_{\varphi}} \approx \frac{16\pi M_{\text{pl}}^2}{m_{\varphi}^3}. \quad (2.4)$$

Combining (2.3) and (2.4) one obtains the reheat temperature in terms of the mass

$$T_{\text{rh2}} \sim m_{\varphi}^{3/2} M_{\text{pl}}^{-1/2}. \quad (2.5)$$

For successful nucleosynthesis one requires the reheating temperature to be greater than a few MeV, and one obtains the famous cosmological moduli problem bound [1–3]

$$m_\varphi \gtrsim 30 \text{ TeV}. \quad (2.6)$$

More detailed cosmological bounds on late time entropy production have been calculated in [4].

Next, we briefly mention the arguments which suggest that cosmologically relevant moduli fields can be expected to be present in supergravity/string constructions with gravity mediation of supersymmetry breaking (though our analysis in no way commits to these models, the bound obtained is equally valid if all the moduli are stabilised supersymmetrically). The form of the supergravity F-term potential

$$V = e^{\mathcal{K}} \left( \mathcal{K}^{ij} D_i W D_j W - 3|W|^2 \right) \quad (2.7)$$

together with the formula for the gravitino mass

$$m_{3/2} = e^{\mathcal{K}/2} |W| \quad (2.8)$$

suggests that once supersymmetry is broken the moduli receive a contribution to their potential which is of the order of the gravitino mass. Furthermore, in gravity mediated supersymmetry breaking the scale of the soft masses is of the order of the gravitino mass. Though there can be exceptions to both these expectations it is reasonable to expect moduli masses at the supersymmetry breaking scale or at most few orders of magnitude higher. For models of low energy supersymmetry, this implies moduli masses at the TeV scale or few orders of magnitude higher in the context of gravity mediated models. There is a tension between the bound (2.6) and supersymmetry as a solution to the hierarchy problem; though sequestering can alleviate the problem.

Recently, cosmology with a modulus/moduli decaying at late times has emerged as the “preferred scenario” in many string constructions. There has been substantial exploration of the associated phenomenology in the context of M-theory compactifications [10, 11]. In these models, the non-thermal dark matter density [12] is of the right order of magnitude if the moduli are at 30-100 TeV. Based on this, the expectation that supersymmetry provides a partial resolution of the hierarchy problem<sup>2</sup> and other phenomenological successes [13] soft masses at 30-100 TeV have been suggested to be a generic prediction of string compactifications [14, 15].

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<sup>2</sup>With soft scalar masses at 100 TeV, one can attribute 14 orders of magnitude of the hierarchy problem (of the total 16) to supersymmetry.

The Large Volume Scenario [16, 17] in type II B provides an explicit realisation of moduli stabilisation. In these models there is one modulus (corresponding to the overall volume of the compactification) which is significantly lighter than all others - it is the last to decay. The models can exhibit sequestering [18–20], there is no theoretical tension with TeV scale supersymmetry. A generic prediction of the cosmology is dark radiation in the form of axions [33, 34]. In addition, the scenario provides non thermal generation of dark matter [23], and its relation to the baryon energy density [24]. For more phenomenological issues, see [25].

### 3 Inflationary Constraints on Light Moduli

In this section we obtain a relationship between the mass of the late decaying modulus field, post-inflationary reheating parameters and inflationary observables. This will involve two steps. First, we obtain an expression for the number of e-foldings in which the universe is matter (modulus) dominated which follows from the evolution of the energy density of the universe from the time of horizon exit to the present day<sup>3</sup>. Then, we express the number of e-foldings in terms of the modulus mass by taking its lifetime to be as given by Eq. (2.4).

Our working assumption will be that the inflaton is responsible for the observed density perturbations; more specifically, none of the light moduli play the role of a curvaton. The history of the universe will be taken to be as described in the introduction and section 2 where reheating phase produced by the inflaton decay is followed by a prolonged matter dominated phase from the coherent oscillation of the modulus field. This scalar field decays before the epoch of big bang nucleosynthesis. We will be explicitly including only one modulus (the one to decay last) in the history.

We characterise the reheating phase by two parameters - the number of e-foldings during the era  $N_{\text{re}}$ , and the equation of state  $w_{\text{re}}$ . Given this general parametrisation we hope to capture not only the transfer of energy from the inflaton to radiation but also of the decay of other moduli (if they are significantly heavier, and decay much earlier) by the “reheating” phase.

We begin the derivation of our relation by writing the condition which determines the exit of a mode of comoving wavenumber  $k$  from the horizon  $k = a_k H_k$  as

$$k = \frac{a_k}{a_{\text{end}}} \cdot \frac{a_{\text{end}}}{a_{\text{re}}} \cdot \frac{a_{\text{re}}}{a_{\text{eq}}} \cdot \frac{a_{\text{eq}}}{a_{\text{decay}}} \cdot a_{\text{decay}} H_k, \quad (3.1)$$

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<sup>3</sup>This can be considered as the generalisation (for late time modulus dominated cosmology) of the equation which gives the total number of e-foldings (see for e.g. [27]) in inflationary models.

where the subscripts end, re, eq and decay indicate the end of inflation, end of reheating, equality of energy density between matter (in this case modulus energy density) and radiation, and decay of the modulus. Taking the logarithm of (3.1) one obtains

$$N_{\text{matdom}} = -N_k - N_{\text{re}} - N_{\text{rad}} - \ln k + \ln(a_{\text{decay}}) + \ln H_k \quad (3.2)$$

where  $N_{\text{matdom}}$  is the number of e-foldings in the matter (modulus) dominated era,  $N_k$  the number of e-foldings between the horizon exit of the mode with mode number  $k$  and end of inflation,  $N_{\text{re}}$  the number of e-foldings during the period of reheating and  $N_{\text{rad}}$  the number of e-foldings in the radiation dominated era.

Next, we obtain another expression for  $N_{\text{matdom}}$  based on the evolution of energy density. We begin by writing<sup>4</sup>

$$-N_{\text{matdom}} = \frac{1}{3} \ln(\rho_{\text{decay}}^{\text{matter}} / \rho_{\text{eq}}^{\text{matter}}). \quad (3.3)$$

The energy density at the time of decay can be expressed in terms of the reheat temperature  $T_{\text{rh2}}$  and the effective number of light species  $g_{\text{rh2}}$  at the time of reheating<sup>5</sup>

$$\rho_{\text{decay}}^{\text{matter}} \approx \rho_{\text{decay}} = (\pi^2/30)g_{\text{rh2}}T_{\text{rh2}}^4, \quad (3.4)$$

and the reheat temperature can be related to the CBM temperature today (assuming dark radiation is insignificant) by

$$T_{\text{rh2}} = (43/11g_{\text{s,rh2}})^{1/3} (a_0/a_{\text{decay}}) T_0 \quad (3.5)$$

where  $g_{\text{s,rh2}}$  is the effective number of light species for entropy. Combining Eq. (3.4) and Eq. (3.5) and expressing  $\ln \rho_{\text{eq}}^{\text{matter}}$  as

$$\ln(\rho_{\text{eq}}^{\text{matter}}) = \ln(\rho_{\text{eq}}^{\text{radiation}}/\rho_{\text{re}}) + \ln(\rho_{\text{re}}/\rho_{\text{end}}) + \ln(\rho_{\text{end}}), \quad (3.6)$$

Eq. (3.3) yields (we take  $\rho_{\text{re}}^{\text{radiation}} \simeq \rho_{\text{re}}$ )

$$\begin{aligned} -\frac{3}{4}N_{\text{matdom}} &= \frac{1}{4} \ln(\pi^2 g_{\text{rh2}}/30) + \frac{1}{3} \ln(43/11g_{\text{s,rh2}}) + \ln(a_0 T_0/a_{\text{decay}}) + N_{\text{rad}} \\ &\quad -\frac{1}{4} \ln(\rho_{\text{end}}) + \frac{3}{4}(1+w_{\text{re}})N_{\text{re}}. \end{aligned} \quad (3.7)$$

Adding (3.2) and (3.7) the dependence on both  $N_{\text{rad}}$  and  $a_{\text{decay}}$  drops out and we obtain

$$\frac{1}{4}N_{\text{matdom}} = \frac{1}{4} \ln(\pi^2 g_{\text{rh2}}/30) + \frac{1}{3} \ln(43/11g_{\text{s,rh2}}) - N_k - \ln(k/a_0 T_0) + \ln H_k$$

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<sup>4</sup>The energy density  $\rho$  with a superscript will denote the energy density in a given form;  $\rho_{\text{decay}}^{\text{matter}}$  is the energy density in the form of matter at the time of modulus decay.

<sup>5</sup> We will explicitly incorporate the phase of final reheating later and find that conclusions are not altered.

$$- \frac{1}{4} \ln(\rho_{\text{end}}) - \frac{1}{4}(1 - 3w_{\text{re}})N_{\text{re}} \quad (3.8)$$

Finally, we express  $N_{\text{matdom}}$  in terms of the modulus mass and lifetime by using the explicit form of the scale factor as a function of time. Recall that if the equation of state is  $w$  the evolution of the scale factor between times  $t_1$  and  $t_2$  is given by

$$(a(t_2)/a(t_1))^{\frac{3}{2}(1+w)} = 1 + \frac{3}{2}(1+w)H(t_1)(t_2 - t_1). \quad (3.9)$$

By demanding that the time elapsed between the end of inflation and the decay of the modulus is the lifetime of the modulus (and assuming  $N_{\text{matdom}} \gg 1$ ;  $N_{\text{re}}, N_{\text{eq}} > 1$ ), we obtain<sup>6</sup>

$$\begin{aligned} \left( \frac{a(t_{\text{decay}})}{a(t_{\text{eq}})} \right)^{3/2} &= \frac{3}{2}H_{\text{eq}}\tau_{\text{mod}} - \frac{3H_{\text{eq}}}{4H_{\text{re}}} \left( \frac{a(t_{\text{eq}})}{a(t_{\text{re}})} \right)^2 - \frac{H_{\text{eq}}}{(1+w)H_{\text{end}}} \left( \frac{a(t_{\text{re}})}{a(t_{\text{end}})} \right)^{(3/2)(1+w)} \\ &= \frac{3}{2}H_{\text{eq}}\tau_{\text{mod}} - \frac{3}{4} - \frac{1}{1+w}e^{-2N_{\text{rad}}} \approx \frac{3}{2}H_{\text{eq}}\tau_{\text{mod}} \end{aligned} \quad (3.10)$$

Thus,

$$N_{\text{matdom}} \approx \frac{2}{3} \log(3/2) + \frac{2}{3} \log(H_{\text{eq}}\tau_{\text{mod}}) \quad (3.11)$$

It can easily be checked that the above (approximate) expression is also correct in the regime  $N_{\text{matdom}} \gg 1$  and  $N_{\text{re}}, N_{\text{rad}} \ll 1$ . Next, we substitute for  $H_{\text{eq}}$  by using Eq. (2.2) and parametrise the initial displacement as  $\varphi_{\text{in}} = Y M_{\text{pl}}$  to obtain

$$N_{\text{matdom}} = -\frac{2}{3} \ln 3 - \frac{5}{3} \ln 2 + \frac{2}{3} \ln m_{\varphi}\tau + \frac{8}{3} \ln Y. \quad (3.12)$$

Equating the two expressions for  $N_{\text{matdom}}$  given by (3.8) and (3.12); and making use of the slow roll expression for the Hubble constant,  $H_k^2 = \frac{1}{2}\pi^2 M_{\text{pl}}^2 r A_s$  (with  $A_s$  the amplitude of scalar fluctuations and  $r$  the tensor to scalar ratio) we get

$$\begin{aligned} \frac{1}{6} \ln m_{\varphi}\tau_{\text{mod}} + \frac{1}{4}(1 - 3w_{\text{re}})N_{\text{re}} + \frac{2}{3} \ln Y &= \frac{1}{4} \ln (\pi^2 g_{\text{rh2}}/30) + \frac{1}{3} \ln (43/11 g_{\text{s,rh2}}) \\ &- \ln (k/a_0 T_0) + \frac{1}{12} \ln(4/3) \\ &- \frac{1}{4} \ln (\rho_{\text{end}}/\rho_{\text{k}}) + \frac{1}{4} \ln (\pi^2 r A_s) - N_{\text{k}} \end{aligned} \quad (3.13)$$

We would like to reiterate that so far no assumption about inflationary physics has been made except for slow-roll. All the inflationary details are encoded in the last three terms of the right hand side of the above equation. We use PLANCK data [27] for quantities that have already been observed with accuracy; the primordial scalar

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<sup>6</sup>We will approximate the evolution by including (in the right hand side Einstein equation) only the dominant component of the energy density in each epoch. We hope to report the results of an exact treatment based on numerics in the future.

amplitude  $\ln(10^{10}A_s) = 3.089$  at the pivot scale  $k = 0.05 \text{ Mpc}^{-1}$  and  $T_0 = 2.725 \text{ K}$ . For the number of degrees of freedom<sup>7</sup>, we use  $g_{\text{rh2}} \approx g_{\text{s,rh2}} \approx 100$ . Moreover, we take  $\tau_{\text{mod}}$  for the modulus as given by (2.4).

Plugging in all this, we find

$$\frac{1}{6} \ln \left( \frac{16\pi M_{\text{Pl}}^2}{m_\varphi^2} \right) + \frac{1}{4}(1 - 3w_{\text{re}})N_{\text{re}} + \frac{2}{3} \ln Y = 55.43 + \frac{1}{4} \ln r - N_k - \frac{1}{4} \ln(\rho_{\text{end}}/\rho_k) \quad (3.14)$$

The above equation is our main result. This equation is the generalisation of the equation giving the number of  $e$ -foldings between horizon exit for the modes relevant for CMB observations and the end of inflation. It can be used to obtain the preferred values of  $N_k$  given the lightest modulus mass.<sup>8</sup>

There are strong reasons to believe (supported by both analytic and numerical work) that the equation of state during reheating satisfies  $w_{\text{re}} < 1/3$  (see for e.g. [27, 28] for discussions). From the point of a scalar field  $\chi$  oscillating about its minimum  $w_{\text{re}} > 1/3$  corresponds to the scalar field potential near its minimum being dominated by higher dimensional operators (greater than  $\chi^6$ ) hence can be considered unnatural<sup>9</sup>. Guided by this we take  $w_{\text{re}} < 1/3$  – the second term in the left hand side is positive definite. Now, the above equation can be easily converted to an lower limit for the mass of the modulus<sup>10</sup>

$$m_\varphi \gtrsim \sqrt{16\pi} M_{\text{Pl}} Y^2 e^{-3(55.43 - N_k + \frac{1}{4} \ln(\rho_k/\rho_{\text{end}}) + \frac{1}{4} \ln r)}. \quad (3.15)$$

We note that even if we consider highly exotic reheating  $w_{\text{re}} > 1/3$ , (3.13) predicts values  $m_\varphi$  to be quite large for  $N_k \approx 50$ ; as long as the number of  $e$ -foldings during reheating is not comparable to the number of  $e$ -folding of modulus domination (as is expected for a light modulus). We will discuss this later and focus on (3.15) for now. Recall also that  $Y$  is the initial field displacement of the light modulus in Planck units. As discussed in section 2 the generic expectation for the initial displacement is of the order of  $M_{\text{Pl}}$ , it cannot affect the value of the right hand side of (3.15) significantly. We note in passing that in deriving the CMP bound one also makes use of the fact that  $Y$  is not expected to be significantly less than one. We emphasise that we have been conservative in estimating the bound; a long reheating phase would make it stronger. When the final reheating phase from the decay of the modulus field is carefully considered in this

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<sup>7</sup>The dependence on the number of degrees of freedom appears as  $\ln(g_{\text{rh2}}^{1/4}/g_{\text{s,rh2}}^{1/3})$ , hence is quite mild.

<sup>8</sup>We note that for  $m_\varphi \approx 100 \text{ TeV}$  and a typical value of  $Y$  ( $Y \approx 1/10$ ); the central value of  $N_k \approx 45$ .

<sup>9</sup>Although  $w_{\text{re}} > 1/3$  cannot be excluded, [29] provided a model.

<sup>10</sup>We note that if the reheating phase is almost instantaneous i.e  $N_{\text{matdom}} \gg N_{\text{re}}$  then the condition  $w_{\text{re}} < 1/3$  is not necessary.



picture, an extra term similar to the 2nd term in Eq. (3.14) appears that involves  $w_{\text{re}2}$  and  $N_{\text{re}2}$  of the final reheating epoch. Following similar arguments as outlined above, the bound remains unchanged.

We would like to briefly comment on the multiple modulus case. As mentioned earlier, given the general parametrisation of the reheating phase, the dynamics of heavier moduli which decay very early on should be captured in the “reheating phase”. The relevant dynamics involves epochs of matter domination and radiation domination, should satisfy the bound  $w_{\text{re}} < 1/3$ . If there are  $N$  moduli at the same mass scale (with a diagonal Kahler metric or if we make the assumption that the Kahler metric is generic as in [26]) then the energy density at equality (2.2) scales as  $N^4$  (for fixed  $\varphi_{\text{in}}$ ); the bound becomes stronger by a factor of  $N$ .

Before studying the bound in the context of various inflationary scenarios, we make a few comments

- Larger the number of e-foldings, stronger the bound. Typically

$$N_{\text{k}} \approx \frac{\beta}{1 - n_s} \quad (3.16)$$

with  $\beta$  a model dependent constant [30, 31]. The bound is highly sensitive to the spectral tilt.

- The second parameter in the exponent,  $\frac{1}{4} \ln(\rho_{\text{k}}/\rho_{\text{end}})$  is positive definite. A large ratio between the energy density at the time of horizon exit and the end of inflation weakens the bound.
- The third parameter  $\frac{1}{4} \ln r$  is negative definite. By itself, this term would strengthen the bound for low values  $r$ .

One should be careful while using the above points to estimate the bound. Given an inflationary model the three parameters in the exponent will not be independent of each other. But as we will see next, without committing much to a particular model of inflation, we can obtain a good understanding based on the class of models.

### 3.1 Small field models

In this class of models, the field variation is sub-Planckian in a typical plateau like potential. The change in energy density between horizon exit and end of inflation is not significant for this kind of models. In this case, it is reasonable to drop the term involving the logarithm of the two energy densities in the exponent of (3.15). Taking

$r = 0.01$ , we get

$$m_\varphi \gtrsim \sqrt{16\pi} M_{\text{pl}} Y^2 e^{-3(54.28 - N_k)} \quad (3.17)$$

To get a feel for the numbers,  $N_k = 50$  and taking  $Y = 1/10$  (in what follows, we will always take  $Y = 1/10$  while quoting numbers) we get

$$m_\varphi \gtrsim 4.5 \times 10^8 \text{ TeV} \quad (3.18)$$

which is well above the bound (2.6) given by the cosmological moduli problem.

Now, we comment on the case of non-generic reheating ( $w_{\text{re}} > 1/3$ ). A useful parametrisation of the length of reheating phase can be done by defining a constant  $\lambda$  by  $N_{\text{re}}(1 - 3w_{\text{re}}) \approx -\lambda N_{\text{matdom}}$ . Even if we take a long exotic reheating phase with  $\lambda \approx 1/3$ , a direct application of (3.14) gives (for  $N_k \approx 50$ ) the mass to be

$$m_\varphi \approx 10^6 \text{ TeV}. \quad (3.19)$$

This is again well above the cosmological moduli problem bound.

Given an inflationary model,  $N_k$  can be determined from precise measurements of  $n_s$  via a relation of the form (3.16). A survey of the values of  $\beta$  associated with various models is given in [30,31]. For a typical value models  $\beta \approx 2$ , and  $n_s = 0.9603$  as given by the central value of PLANCK;  $N_k \approx 50$ . The bound in this case is  $4.5 \times 10^8 \text{ TeV}$ . Of course, given the form of (3.16) a small variation in  $n_s$  can lead to a large change in  $N_k$ ; this can alter the bound significantly given the exponential dependence. At  $1\sigma$ , the variation of the spectral index for PLANCK is  $\Delta_{1\sigma} n_s = 0.0073$ . The  $1\sigma$  upper limit of  $n_s$  gives  $m_\varphi > M_{\text{pl}}$ , ruling out late time modulus cosmology (for small field models with  $\beta = 2$ ,  $r = 0.01$ ). On the other hand the lower value gives  $m_\varphi \gtrsim 0.1 \text{ TeV}$ , two to three orders of magnitude below the one given by the cosmological moduli problem. Given this sensitivity<sup>11</sup>, future experiments [35–38] which will lower the uncertainties in the measurement of  $n_s$  by one order of magnitude will play an important role (the uncertainty in the mass will be reduced to two orders of magnitude) in determining the existence of late time modulus decay cosmology in this context.

## 3.2 Large Field Models

As prototypes of the large fields models we consider models where the inflation potential is a monomial  $V_\chi = \frac{1}{2} m^{4-\alpha} \chi^\alpha$  (keeping the “ $m^2 \chi^2$ ” model [39–41] and axion

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<sup>11</sup>Theoretically, the exponential sensitivity implies that  $1/N_k$  corrections to (3.16) can be relevant in some models. We have computed the leading corrections to (3.20) for monomial potentials and found that they do not significantly alter the bound.

monodromy [42, 43] models in mind). For these models

$$n_s - 1 = -(2 + \alpha)/(2N_k), \quad r = 4\alpha/N_k. \quad (3.20)$$

Thus measurement of  $n_s$  and  $r$  determines both  $N_k$  and  $\alpha$ . But given the uncertainty in the measurements of  $r$ , we will take observational input only from  $n_s$ . We will treat  $\alpha$  as a model building parameter –  $N_k$  and  $r$  will be treated as the derived quantities in (3.20).

Let us evaluate the exponent in (3.15). From (3.20) we have  $N_k = (2 + \alpha)/2(1 - n_s)$ ,  $r = 8\alpha(1 - n_s)/(\alpha + 2)$ . To compute  $\log(\rho_k/\rho_{\text{end}})$  we need to express the energy densities at the time of horizon exit and end of inflation in terms of  $n_s$  and  $\alpha$ . The energy density at the time of horizon exit is simply the value of the potential at the time of horizon exit,  $\rho_k = \frac{1}{2}m^{4-\alpha}\chi_k^\alpha$ . The displacement at the time of horizon exit is given by  $\chi_k^2 = 2\alpha M_{\text{pl}}^2 N_k$ . On the other hand, at the end of inflation the energy density is given by  $\rho_{\text{end}} = (1 + \lambda)V_{\text{end}} = \frac{1}{2}(1 + \lambda)m^{4-\alpha}\chi_{\text{end}}^\alpha$ ; where  $\lambda = (1 + 2/\epsilon_0)^{-1}$  with  $\epsilon_0$  the value of the slow roll parameter  $\epsilon$  at the end inflation ( $\epsilon_0 \approx 1$ ). The field displacement at the end of inflation is given by  $\chi_{\text{end}}^2 = (\alpha^2 M_{\text{pl}}^2/2\epsilon_0)$ . Combining the above, we find

$$-N_k + \frac{1}{4} \ln(\rho_k/\rho_{\text{end}}) + \frac{1}{4} \ln r = -\frac{(2 + \alpha)}{2(1 - n_s)} - \frac{1}{4} \ln 3 + \frac{1}{8}(\alpha + 8) \ln 2 + \frac{1}{8}(\alpha - 2) \ln \left( \frac{2 + \alpha}{\alpha(1 - n_s)} \right); \quad (3.21)$$

the bound becomes

$$m_\varphi \gtrsim \sqrt{16\pi} M_{\text{pl}} Y^2 e^{-3 \left( 55.85 - \frac{(2+\alpha)}{2(1-n_s)} + \frac{\alpha}{8} \ln 2 + \frac{1}{8}(\alpha-2) \ln \left( \frac{2+\alpha}{\alpha(1-n_s)} \right) \right)} \quad (3.22)$$

The coefficients of the logarithmic terms are such that not much error is made if one drops the terms; we write

$$m_\varphi \gtrsim \sqrt{16\pi} M_{\text{pl}} Y^2 e^{-3 \left( 55.85 - \frac{(2+\alpha)}{2(1-n_s)} \right)} \quad (3.23)$$

For the  $\frac{1}{2}m^2\chi^2$  and  $n_s$  as given by the central value of PLANCK the bound is  $m_\varphi \gtrsim 10^7$  TeV, one order of magnitude below (3.18). On the other hand for  $\alpha = 1$  and  $n_s$  at the same value the bound becomes  $m_\varphi > 10^{-10}$  TeV, which is completely irrelevant.

To summarise, for  $N_k \approx 50$ , the bound is significantly stronger than that provided by the cosmological moduli problem, and larger the  $N_k$ , stronger the bound is. From the point of view of microscopic models, there is a high sensitivity on the parameters in the inflaton potential as a result of the exponential dependence on  $N_k$  and the fact that  $N_k$  scales as the inverse of the spectral tilt. The parameter  $\beta$  (as defined in (3.16)) plays a central role in determining the magnitude of the bound;  $\beta \approx 2$  (which seems to be the typical value, see for e.g. [31]) and  $n_s$  at the central value of PLANCK correspond to  $N_k \approx 50$  and hence imply a strong bound.

For the non-generic case of exotic reheating with  $w_{\text{re}} > 1/3$ , it is useful to parametrise the duration of reheating by a parameter  $\lambda$ ;  $N_{\text{re}}(1 - 3w_{\text{re}}) = -\lambda N_{\text{matdom}}$ . Even for  $\lambda = 1/3$  (which corresponds to a rather long phase of exotic reheating) direct use of (3.13) gives (for  $N_k \approx 50$ )  $m_\varphi \approx 10^6$  TeV. Again, well above the CMP bound.

As discussed earlier, we have taken moduli interactions to be Planck suppressed in obtaining (3.15). In string constructions of brane world models there can be moduli whose interactions are not Planck suppressed but by a scale<sup>12</sup>  $\Lambda$ . If such a modulus ( $\chi$ ) is the last to decay the CMP bound [1–3] gets modified to The reheat temperature after the decay of a modulus is given by  $T_{\text{reheat}} \sim \sqrt{\Gamma M_{\text{pl}}}$ , where  $\Gamma$  is the width of the modulus. The characteristic width of a modulus  $\chi$  whose interactions in the four dimensional effective action are suppressed by a scale  $\Lambda$  are given by

$$\Gamma_\Lambda \approx \frac{16\pi m_\chi^3}{\Lambda^2} \quad (3.24)$$

Combining the above with the requirement of a sufficiently high reheat temperature for nucleosynthesis one arrives at a generalisation of the CMP bound [1–3] discussed in the introduction

$$m_\chi \gtrsim \eta^{2/3} .30 \text{ TeV} \quad (3.25)$$

where  $\eta = \Lambda/M_{\text{pl}}$ .

Following the same steps as in the earlier part of the section (while using the lifetime of the modulus to be as given by (3.24)), one can obtain the modification of our bound

$$m_\varphi \gtrsim \sqrt{16\pi} M_{\text{pl}} \eta Y^2 e^{-3\left(55.43 - N_k + \frac{1}{4} \ln\left(\frac{\rho_k}{\rho_{\text{end}}}\right) + \frac{1}{4} \ln r\right)}. \quad (3.26)$$

We note that both (3.25) and (3.26) scale as a positive power of  $\eta$ . Carrying out a analysis as above, one easily sees that our bound is stronger in a large range of the phenomenologically interesting parameter space.

Finally, we would like to briefly discuss the cases in which the modulus primarily decays to massive particles. Such decay products can be super partners of standard model particles or additional Higgses. For models in which the primary mode of decay is to massive particles and the lifetime scales as  $m_\varphi^p$  with  $p \leq -1$  our analysis will provide a lower bound on moduli masses. The bound might involve the mass of the decay products (expression for the bound will in general be different from that given in Eq. (3.17) and Eq. (3.22)). In a large number of situations, the lifetime has the same

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<sup>12</sup> The scale  $\Lambda$  can be lower than the Planck scale for the modulus which parametrises the size of the cycle that the branes wrap. In this case  $\Lambda$  is the string scale (see for e.g. [32]). We note that there is a large difference between the string and Planck scale only if the volume of the compactification is large.

form as (2.4) or has the form (see for e.g [33, 34, 44])

$$\tilde{\tau} \approx \frac{16\pi M_{\text{pl}}^2}{m_\varphi \tilde{m}^2} \quad (3.27)$$

(i.e.  $p = -1$ ) where  $\tilde{m}$  is the mass of the decay products. Again, following the same steps as in the earlier part of the section one obtains a bound on the mass of the decay products (in the case that the lifetime takes the form (3.27)) or a bound on the modulus mass (in the case that the lifetime takes the same form as (2.4)). But, a bound on the mass of the decay products translates to a bound on the mass of the modulus, as the mass of the modulus has to be heavier than the mass of the decay products. Thus, the bound (3.15) applies equally well for these situations (with  $p = -1$ ). On the other hand, for  $p > -1$  our analysis will provide an upper bound for moduli masses (in terms of the mass of the decay products). This can be very interesting, although such models are not generic. We leave the detailed study of specific models for future work. In addition, thermal bath produced by the reheating may have some effects on the decay rate of the moduli [45], thus on our bound. We will address the issue in future work.

## 4 Conclusions

We have considered cosmologies in which density perturbations are generated by the quantum fluctuations of the inflaton field at early times; the late time dynamics involves a modulus which first dominates the energy density of the universe and then decays to reheat the visible sector. In this context we have obtained a relationship between the mass of modulus, broad characteristics of post-inflationary reheating and inflationary observables (or parameters in the inflaton potential). Together with the bound  $w_{\text{re}} < 1/3$  on the equation of state during reheating (or if reheating is almost instantaneous) and the generic expectations on the initial displacement of the modulus at the end of inflation ( $\varphi_{\text{in}} \sim M_{\text{pl}}$ ) the relation gives a bound on the minimum mass of the modulus, see Eq. (3.15). For a large class of inflationary models the bounds obtained (for values of  $n_s$  at the central value of PLANCK) are much stronger than the “cosmological moduli problem” bound. The bound is exponentially sensitive to  $N_k$  and future observations will play an important role in sharpening the bound. In fact, with precise measurements of inflationary observables it is possible that the bound becomes highly constraining.

A typical value of  $N_k \approx 50$  suggests a very high value for the modulus mass in the context of late time modulus dominated cosmology. Given a particular model of inflation,  $N_k$  is known in terms of observable parameters. Larger the value  $N_k$ , more severe is the bound. In the case of instantaneous reheating, the bound becomes an equality and gives

a prediction for the mass of the modulus. Even if we take  $w_{\text{re}} > 1/3$  (an exotic reheating phase); the value of the modulus mass obtained from our analysis (for  $N_k \approx 50$ ) is much higher than the CMP bound as long as the number of e-foldings during reheating are not comparable to the number of e-foldings during modulus domination (as is expected for a light modulus).

The bound should have broad implications for string/supergravity models where it is typical to have scalars interacting with Planck suppressed interactions. It can shed light on the scale of supersymmetry breaking in the context of gravity mediated breaking; where the scale of soft masses can be tied to the moduli masses. The bound is exponentially sensitive to the number of e-foldings during inflation and hence provides a new motivation for precision measurements of the spectral tilt.

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